Eur. Phys. J. A **18**, 155–158 (2003)
DOI 10.1140/epja/i2002-10291-5

THE EUROPEAN PHYSICAL JOURNAL A

Relativistic study of nucleon electroweak properties in a constituent-quark model

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Received: 30 September 2002 /
Published online: 22 October 2003 –

Abstract. We discuss the predictions of the Goldstone-boson-exchange constituent-quark model for the proton and neutron electric and magnetic form factors as well as the nucleon axial and induced pseudoscalar form factors. The results are calculated in a covariant framework using the point-form approach to relativistic quantum mechanics. The only input into the calculations is the nucleon wave function from the constituent-quark model. A remarkably consistent picture, with all aspects of the electroweak nucleon structure close to existing experimental data, is obtained.

PACS. 12.39.-x Phenomenological quark models – 13.66.-a Lepton-lepton interactions – 14.20.Dh Protons and neutrons

To treat a system of several particles relativistically, one must employ a quantum theory fulfilling all the symmetry requirements imposed by the Poincaré group. According to Dirac [1] there exist three possibilities for a relativistic quantum theory such that the generators of the Poincaré group are minimally affected by interactions: the instant form, the front form, and the point form.

In the point-form approach only the components of the four-momentum operator $\hat{P}^\mu$ contain interactions. Moreover, all Lorentz transformations (in particular the Lorentz boosts) remain purely kinematic so that the point form is manifestly covariant.

To construct the interacting four-momentum operators for a quantum theory with a fixed number of particles (as in a constituent-quark model) we use the Bakamjian-Thomas (BT) construction [2]. Starting from the free generators of the Poincaré group one introduces the auxiliary operators $\hat{M}_{\rm fr} = \sqrt{\hat{P}^\mu_{\rm fr}\hat{P}_{{\rm fr}\,\mu}}$ (free mass operator) and $\hat{V}^\mu_{\rm fr}$ (free four-velocity operator) defined by $\hat{M}_{\rm fr}\hat{V}^\mu_{\rm fr} = \hat{P}^\mu_{\rm fr}$, where $\hat{P}^\mu_{\rm fr}$ is the free four-momentum operator. Thereby all the dynamics is introduced into the mass operator (the four-velocity operator remaining unaffected by interactions), and the four-momentum operator can be written as

$$\hat{P}^\mu := \hat{P}^\mu_{\rm fr} + \hat{P}^\mu_{\rm int} = \hat{M}\hat{V}^\mu_{\rm fr} = (\hat{M}_{\rm fr} + \hat{M}_{\rm int})\hat{V}^\mu_{\rm fr}. \qquad (1)$$

Poincaré covariance requires that $\hat{M}$ commutes with $\hat{V}^\mu_{\rm fr}$ and is a scalar under Lorentz transformations. Eigenstates of the four-momentum operator arise as simultaneous eigenstates of the mass and the velocity operators. The motion of the system as a whole and the internal motion are separated. The latter is described by a wave function of internal degrees of freedom which is obtained by solving the eigenvalue problem for the mass operator

$$\hat{M}\Psi = M\Psi. \qquad (2)$$

For a constituent-quark model (CQM) of baryons the Hilbert space is spanned by product states of three spin-$\frac{1}{2}$ particles

$$|p_1,p_2,p_3;\sigma_1,\sigma_2,\sigma_3\rangle = |p_1,\sigma_1\rangle \otimes |p_2,\sigma_2\rangle \otimes |p_3,\sigma_3\rangle\,. \qquad (3)$$

In the point form, however, a more useful basis is provided by the so-called velocity states [3]

$$|v;\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3;\mu_1,\mu_2,\mu_3\rangle = U_{B(v)}\,|k_1,k_2,k_3;\mu_1,\mu_2,\mu_3\rangle = \prod_{i=1}^{3} D^{1/2}_{\sigma_i\mu_i}[R_{\rm W}(k_i,B(v))]\,|p_1,p_2,p_3;\sigma_1,\sigma_2,\sigma_3\rangle\,. \qquad (4)$$

Here, $B(v)$ is a boost with four-velocity v and $U_{B(v)}$ its unitary representation, $p_i = B(v)k_i$, where the $k_i = (\omega_i,\boldsymbol{k}_i)$ are momenta that are restricted by $\sum \boldsymbol{k}_i = 0$, and $D^{1/2}$ are the spin-1/2 representation matrices of Wigner

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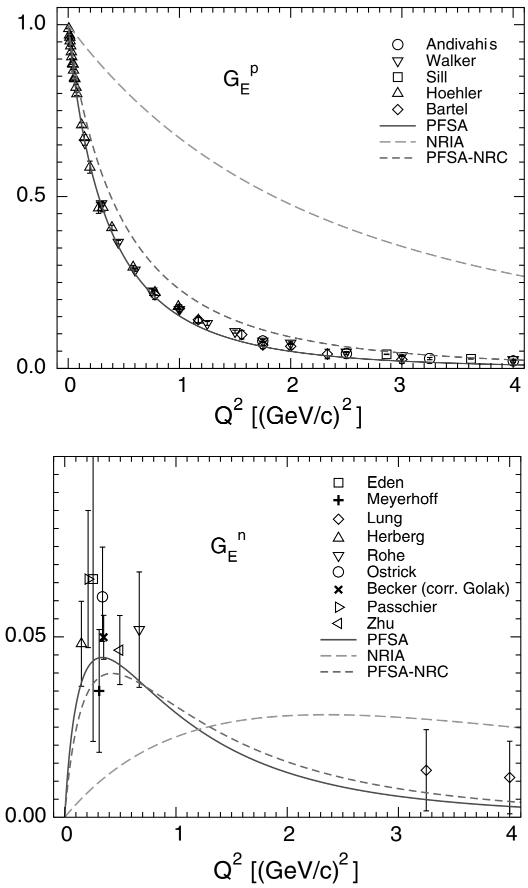

Fig. 1. Proton and neutron electric form factors.

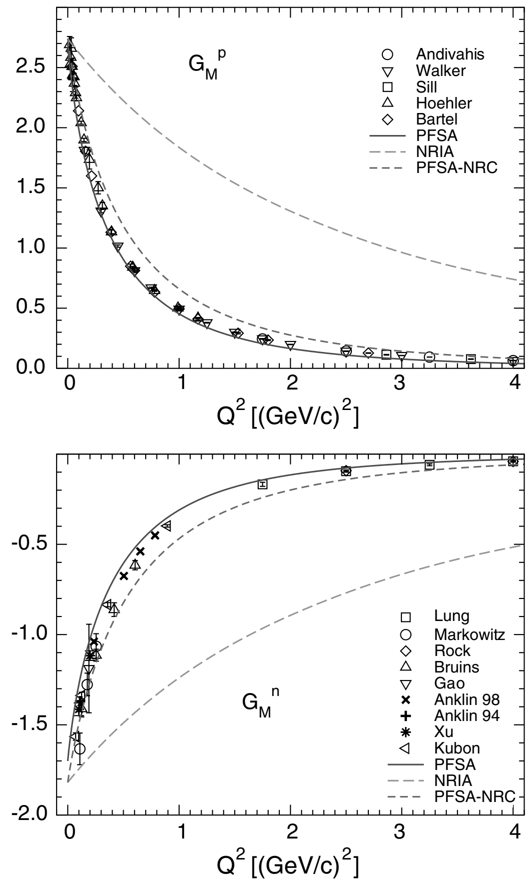

Fig. 2. Proton and neutron magnetic form factors.

rotations $R_{\mathrm{W}}(k_i, B(v))$. The velocity states are simultaneous eigenstates of the free mass and four-velocity operators. The baryon wave function is the velocity state representation of the mass operator eigenstate $|P, \Sigma\rangle$, where P and Σ are the eigenvalues of the total momentum and the z-component of the total angular momentum, respectively.

In a recent CQM [4] the interaction between two constituent quarks i, j was based on Goldstone-boson-exchange (GBE) dynamics in addition to a linear confinement. The corresponding mass operator is given by

$$\hat{M} = \sum_{i=1}^{3} \sqrt{\hat{m_i^2} + \hat{\boldsymbol{k}_i^2}} + \sum_{i<j} \left[\hat{V}_{\mathrm{conf}}(i,j) + \hat{V}_{\mathrm{GBE}}(i,j) \right] . \quad (5)$$

Even though $\hat{V}_{\mathrm{conf}}$ and $\hat{V}_{\mathrm{GBE}}$ are nonrelativistic potentials, we emphasize that this mass operator fulfills all the requirements of Poincaré covariance.

The electromagnetic and axial form factors of the nucleons are related to matrix elements of electromagnetic and axial current operators between states $|P, \Sigma\rangle$ and $|P', \Sigma'\rangle$ of the incoming and outgoing nucleon. For the current operator we apply the point-form spectator approximation (PFSA) [3]. It relies on the assumption that two constituent quarks are spectators in the sense that

$$\begin{aligned} &\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' | \, \hat{J}^\mu(0) \, | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle = \\ &2E_2 \delta(\boldsymbol{p}_2' - \boldsymbol{p}_2) \delta_{\sigma_2' \sigma_2} 2E_3 \delta(\boldsymbol{p}_3' - \boldsymbol{p}_3) \delta_{\sigma_3' \sigma_3} \\ &\quad \times \langle p_1', \sigma_1' | \, \hat{j}^\mu(0) \, | p_1, \sigma_1 \rangle \ , \end{aligned} \quad (6)$$

and similarly for the axial current with $\hat{J}^\mu(0)$ replaced by $\hat{\boldsymbol{A}}^\mu(0)$ and $\hat{j}^\mu(0)$ by $\hat{\boldsymbol{a}}^\mu(0)$. Here, we indicate one-body currents of the constituent quarks by small letters. The axial current is a vector in isospin space. It is important to notice that the impulse delivered to the constituent quark, $\tilde{q} = p_1' - p_1$, is different from the impulse delivered to the nucleon, $q = P' - P$. The momentum transfer $\tilde{q}$ is uniquely determined by q and the two spectator conditions $\boldsymbol{p}_2' = \boldsymbol{p}_2$ and $\boldsymbol{p}_3' = \boldsymbol{p}_3$. For the remaining one-body current matrix element we employ the familiar expressions for the electromagnetic and axial currents of pointlike spin-$\frac{1}{2}$ particles

$$\langle p_1', \sigma_1' | \, \hat{j}^\mu(0) \, | p_1, \sigma_1 \rangle = \bar{u}(p_1', \sigma_1') \gamma^\mu e_1 u(p_1, \sigma_1), \quad (7)$$

$$\begin{aligned} &\langle p_1', \sigma_1' | \, \hat{\boldsymbol{a}}^\mu(0) \, | p_1, \sigma_1 \rangle = \\ &\quad \bar{u}(p_1', \sigma_1') \left[\gamma^\mu + \frac{2 f_\pi}{\tilde{Q}^2 + \mu_\pi^2} g_{\pi q} \tilde{q}^\mu \right] \gamma_5 \frac{\boldsymbol{\tau}_1}{2} u(p_1, \sigma_1). \end{aligned} \quad (8)$$

In the electromagnetic current, e_1 is the charge of the constituent quark. In the first term of the axial current we

Table 1. Proton and neutron electric radii (in fm^2) and magnetic moments (in n.m.).

	Experiment	PFSA	NRIA	PFSA-NRC
r_p^2	0.757(14) [9]	0.82	0.10	0.57
r_n^2	−0.1161(22) [9]	−0.133	−0.009	−0.093
μ_p	2.792847337(29) [9]	2.70	2.74	2.74
μ_n	−1.91304270(5) [9]	−1.70	−1.82	−1.82

assume that the axial coupling of the constituent quark is equal to one as for a free bare fermion. The second term is due to the coupling of the pion to the constituent quark. It is proportional to the pion decay constant $f_\pi = 93.2$ MeV and the pion-quark coupling $g_{\pi q}$. For the latter we take $\frac{g_{\pi q}^2}{4\pi} = 0.67$; this is the same value as in the parametrization of the pion-exchange potential of ref. [4]; μ_π is the pion mass and $\tilde{Q}^2 = -\tilde{q}^\mu \tilde{q}_\mu$.

The predictions of the GBE CQM for the nucleon electromagnetic and axial form factors calculated in PFSA have been obtained in refs. [6–8]. Figures 1 and 2 show the electric and magnetic form factors of the proton and the neutron. The PFSA results (solid lines) fall rather close to the experimental data, while the predictions in nonrelativistic impulse approximation (NRIA) represented by the long-dashed lines deviate drastically. The short-dashed lines (labelled by PFSA-NRC) are the results of a calculation with a nonrelativistic current but with boosts included. One observes that the inclusion of boost effects brings the results already near to the PFSA predictions. Therefore, the main part of the difference between PFSA and NRIA is due to relativistic boosts, and effects from the relativistic current operator are of secondary importance.

The electric radii and magnetic moments of proton and neutron are given in table 1. The electric radii show the same characteristics as just outlined for the form factors. For the magnetic moments it appears at first instance that a nonrelativistic theory would be appropriate. A closer inspection, however, reveals that the PFSA results are governed by considerably large relativistic effects both from the relativistic current operator and from Lorentz boosts. In NRIA, *i.e.* using a nonrelativistic current operator, boosts have no influence, and incidentally a reasonable result is recovered, even though a covariant theory suggests large relativistic effects also for the magnetic moments.

In figs. 3 and 4 we show analogous results for the axial and induced pseudoscalar nucleon form factors. While the PFSA predictions are found in reasonable agreement with experimental data, the NRIA results appear again unacceptable. The dash-dotted curve in fig. 3 represents the result of a calculation with a relativistic current but without Lorentz boosts; it remains far away from a reasonable description. For the induced pseudoscalar form factor in fig. 4 the inclusion of the pion pole term for the axial current in eq. (8) is essential. The axial coupling constant (axial charge) $g_A = G_A(Q^2 = 0)$ is not affected by Lorentz boosts. The covariant result (*i.e.* using a relativistic current) is 1.15, thus slightly smaller than the experimental value 1.2670(30), while the nonrelativistic result of 1.65 is too large.

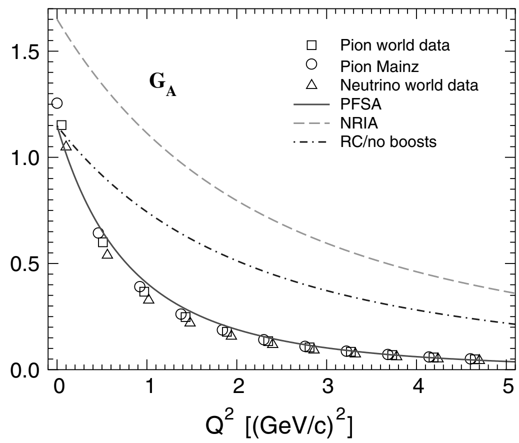

Fig. 3. Nucleon axial form factor.

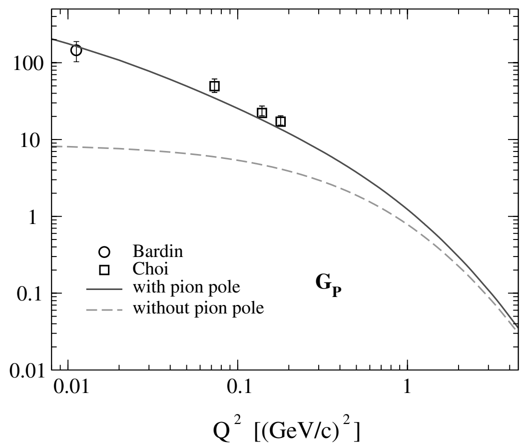

Fig. 4. Nucleon induced pseudoscalar form factor.

In summary the PFSA predictions for all electroweak nucleon form factors fall rather close to the experimental data. The nonrelativistic results deviate drastically. The inclusion of Lorentz boosts is most essential. Obviously, a reasonable description of the electroweak nucleon structure requires a covariant theory.

This work was supported by the Austrian Science Fund, Project No. P14806.

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